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To: Dr. Don Mills
Director of Research

UNAVAILABILITY OF THE RUPTURE PANELS AT PICKERING NGS

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The calculation of the size of an acceptance sample plan and a method for estimating the unavailability of the rupture panel to be installed at Unit 2 of Pickering Nuclear Generation Station is presented. It is shown that the safety requirements are met, with 99% confidence, if 48 panels are tested and no failure is observed.

1. INTRODUCTION

This study was carried out in response to an inquiry by the Nuclear Systems Department to (i) determine the size of an acceptance sample plan for rupture panels to be installed at Pickering NGS A as part of the new rupture panel system; and (ii) evaluate the unavailability of the rupture panels based on the sample results.

The rupture panel system (RPS) is part of a program presently being implemented in the four units at Pickering NGS A. This system will replace the existing vapor barriers and louvers at each unit. The primary purposes of the system are: (i) to isolate the non-accident units from the pressure relief duct (PRD); and (ii) to relieve the pressure from a reactor building (RB) in case of break in the heat transport system or in the secondary side.

As part of the RPS, a total of 29 rupture panels will be installed in each RB (see Figure 1). Each rupture panel will have a relief area of 0.63 m^2 and is designed to open at a pressure of 15 ± 2.5 kpa (1.8 to 2.5 psi). The panels will be mounted on a bulkhead between the PRD and the RB and are intended to open in case of a loss of coolant accident (LOCA) or a main steam line break accident (MSLB) to relieve the pressure within the RB. For a LOCA an area of 6 m^2 is required for pressure relief and for a MSLB an area of 13 m^2 (see /1/). These imply that at least 10 panels must burst in case of a LOCA and 21 in case of a MSLB. Target unavailabilities for the rupture panels have been set at 1 X 10^{-4} for a LOCA and at 9 x 10^{-4} for a MSLB.

2. PROPOSED METHOD

2.1 Unavailability of the Panels

To calculate the unavailability of the rupture panels, it is reasonable to assume that the panels are indistinguishable and that all have the same (unknown) probability, p, of failure, i.e., of not bursting. Denote by n the number of panels to be installed, and by m the number

of panels required to burst in a given type of accident. The unavailability of the panels for that accident is given by the probability that less than m out of the n panels burst at a given time. This unavailability is a function of p and is given by

$$\text{UNAV}(p) = \sum_{l=0}^{m-1} \binom{n}{l} p^{n-l} (1 - p)^{l}. \quad (1)$$

The complement of this probability (availability of the panels) is the probability that at least m out of n panels burst and is given by

$$\text{AVAL}(p) = 1 - \text{UNAV}(p). \quad (2)$$

Figure 2 presents a plot of this function for $n = 29$ and $m = 21$ where it can be observed that this is a decreasing function of p.

In general p is not known and has to be estimated. One way of doing it is to take a sample of N panels and measure the bursting pressure of each panel. A panel counts as a success if its bursting pressure lies within the interval 15 ± 2.5 kpa, otherwise it counts as a failure. If F failures are observed in the sample, p is estimated as

$$\hat{p} = F/N.$$

The values of UNAV and AVAL are estimated by substituting this value in equations (1) and (2).

In many cases a point estimate is not as important as to determine a confidence interval for p. The confidence interval is constructed so that it contains the true value of the parameter with a pre-established confidence or probability. For example, a 99% (upper) confidence interval for p will be of the form $[0, p_u]$ where p_u is chosen so that, if repeated samples were taken, the interval would contain the true value of p an average of 99 out of 100 times. The value of p_u can be evaluated in each sample as the solution to the equation/2/

$$1 - c = \sum_{j=0}^{F} \binom{n}{j} p_u^{j} (1 - p_u)^{n-j} \quad (1)$$

where c is the confidence value for the interval. Note that p_u can be interpreted as the largest estimate for p consistent with the data at the confidence level c. The interval $[\text{AVAL}(p_u), 1]$ will give a $100c\%$ (lower) confidence interval for AVAL(p), whereas $[0, \text{UNAV}(p_u)]$ will give and (upper) interval for the unavailability of the panels.

To illustrate the different estimators, consider a scenario where 60 panels are tested and no failures are obtained. The estimate $\hat{p}$ will be equal to zero (i.e. all the panels would be assumed to work) and the corresponding estimate for AVAL(p) will be equal to one (i.e. it will certainly occur). These are not realistic estimates. Better estimates are obtained using confidence intervals. Table 1 gives the values of p_u and $\text{UNAV}(p_u)$ (per reactor and for the four reactors) at different confidence levels. Note the effect of an increase of the confidence level on the size of the interval. For example, an increase from 98% to 99% on the confidence level, more than triples the size of $\text{UNAV}(p_u)$ (ie., p_u increased only by 17%).

2.2 Sample Design

To design a sample acceptance plan it is necessary to select the sample size, N, and the maximum number of defective panels that can be tolerated, F. N and F have to be selected so

that UNAV(p_u) $\leq 10^{-4}$. Table 2 gives the values of UNAV(p_u) for several values of N and F when $n = 29$ and $m = 21$ and Table 3 when $n = 29$ and $m = 10$. It can be seen that the smallest sample size at which the availability of the system is met, is $N = 48$ (with $F = 0$). If one of the 48 panels fails during the sample, either all panels has to be rejected or extra 21 panels, for a total of 69, have to be tested; this would give a plan with $N = 69$ and $F = 1$. If two fail, then it is recommended to reject all panels because it becomes to expensive to test the 88 panels that would be required to meet the design requirements.

3. CONCLUSION

To determine whether a batch of panels will meet the requirements, a sample plan was designed by requiring, with 99% confidence, that the unavailability targets are met. It requires that a minimum of 48 panels be tested. If no panel fails during the testing, the panels can be accepted. If two or more fail, the batch should be rejected. If only one fails, either the batch should be rejected or 21 extra panels can be tested. If no additional panel fails, the batch can be accepted, otherwise it should be rejected.

The method presented in the report makes only two hypotheses about the nature of the panels, namely, that the panels are indistinguishable and that all have the same probability of failure. These are realistic assumptions and are in keeping with the requirements of the problem. There is another approach which allow additional information about the quality of the panels to be included in the analysis. This is explained in Appendix A.

It is possible to obtain better estimates for p if it can be assumed that the pressure bursting points of the panels have a normal distribution. This hypothesis can be tested if information of the bursting pressure of the panels is recorded during the sampling. It is strongly recommended that this data be gathered. The method is described in Appendix B.

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REFERENCE

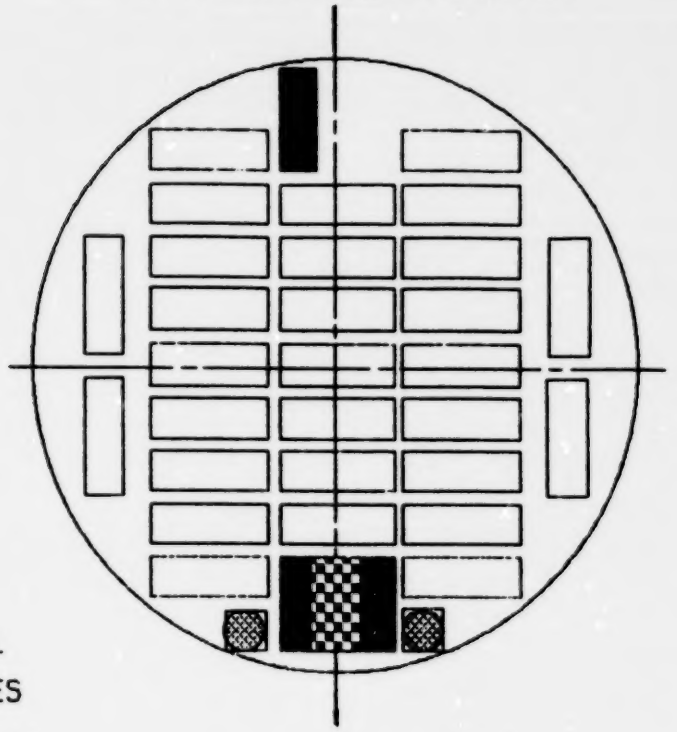
1. Penn, N.J., (1987). "Pickering NGS, Analysis of Safety Design Requirement for the Rupture Panel System." Ontario Hydro NSSD Report No. 87270, to be issued.

2. Lloyd, D.K. and Lipow, M., (1962). *"Reliability: Management, Methods and Mathematics."* Prentice Hall., Englewood Cliffs, New Jersey.

FIGURE 1

PICKERING NGS A

RUPTURE PANEL SYSTEM



Legend

- RUPTURE PANEL
- BY-PASS VALVES
- ACCESS DOORS
- BLANKED OFF AREAS

* TOTAL OF 29 RUPTURE PANELS MOUNTED ON EACH BULKHEAD

* EACH RUPTURE PANEL HAS A RELIEF AREA OF APPROX. 0.63 SQ.M.; TOTAL AREA OF RELIEF IN EACH REACTOR BUILDING IS >18 SQ.M.

* AREA REQUIRED FOR PRESSURE RELIEF:

  (a) LARGE LOCA = 6 SQ.M. (10 PANELS FULLY OPEN)

  (b) MAIN STEAM LINE BREAK = 13 SQ.M. MIN. (21 PANELS FULLY OPEN)

* OVERALL UNAVAILABILITY REQUIRED FOR PR. RELIEF IS 10 - 3 (MSLB)

  BUTTERFLY VALVES CONTRIBUTE < 1 X 10 - 4 (WITH MONTHLY TESTING)

  SO THE PANELS CANNOT EXCEED 9 X 10 - 4 PER BULKHEAD.

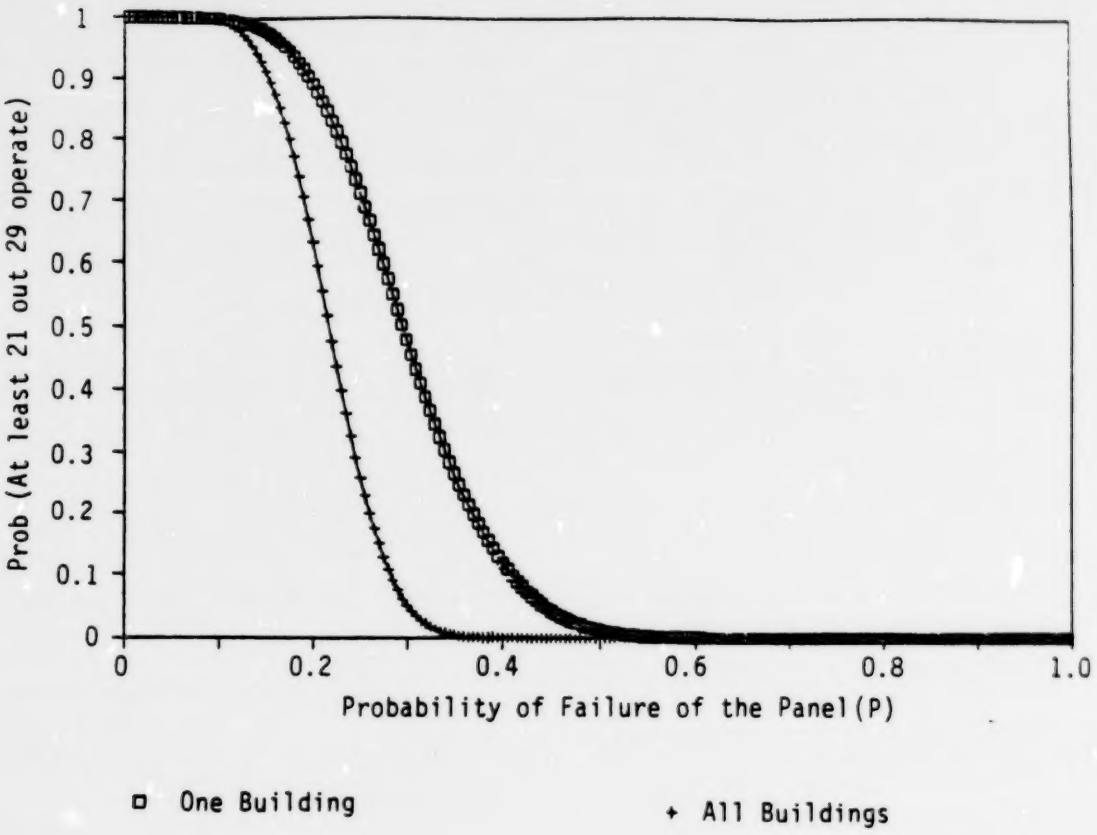


FIGURE 2
RUPTURE PANEL AVAILABILITY AS A FUNCTION OF P

TABLE 1

Interval Estimates for the Probability of Failure of the Panels
and Probability that at least 21 out of 29 panels operate

Number of Panels Initially Tested :	60
Number of Failures :	0
Number of Panels installed per Reactor Building :	29

Confidence Level	p_u	UNAV(p_u) Per Reactor	UNAV(p_u) All Reactors
50% confidence	.115E-01	.000	.000
75% confidence	.228E-01	.000	.000
95% confidence	.487E-01	.632E-05	.253E-04
98% confidence	.631E-01	.498E-04	.199E-03
99% confidence	.739E-01	.168E-03	.670E-03

TABLE 2

UNAVAILABILITY FOR MAIN STEAM LINE BREAKS
(99% Confidence)

Number of Panels installed per reactor building = 29
Number of Panels required to burst per reactor building = 21

N	F=0	F=1	F=2	F=3	F=4	F=5	F=6
10	.800E+00	.989E+00	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01
15	.352E+00	.796E+00	.960E+00	.995E+00	.100E+01	.100E+01	.100E+01
20	.124E+00	.468E+00	.765E+00	.920E+00	.978E+00	.995E+00	.999E+00
25	.435E-01	.235E+00	.502E+00	.731E+00	.877E+00	.951E+00	.983E+00
30	.162E-01	.113E+00	.296E+00	.510E+00	.699E+00	.834E+00	.918E+00
32	.111E-01	.840E-01	.236E+00	.431E+00	.620E+00	.771E+00	.875E+00
34	.774E-02	.628E-01	.187E+00	.361E+00	.544E+00	.703E+00	.823E+00
36	.545E-02	.471E-01	.148E+00	.300E+00	.473E+00	.634E+00	.765E+00
38	.387E-02	.355E-01	.118E+00	.249E+00	.407E+00	.566E+00	.704E+00
40	.278E-02	.269E-01	.932E-01	.205E+00	.349E+00	.501E+00	.642E+00
42	.202E-02	.205E-01	.739E-01	.169E+00	.297E+00	.441E+00	.580E+00
44	.148E-02	.157E-01	.588E-01	.139E+00	.252E+00	.385E+00	.521E+00
46	.110E-02	.121E-01	.469E-01	.114E+00	.214E+00	.335E+00	.465E+00
48	.820E-03	.936E-02	.375E-01	.942E-01	.181E+00	.291E+00	.413E+00
50	.617E-03	.730E-02	.301E-01	.776E-01	.153E+00	.252E+00	.365E+00
51	.537E-03	.645E-02	.270E-01	.705E-01	.140E+00	.234E+00	.343E+00
52	.469E-03	.572E-02	.242E-01	.640E-01	.129E+00	.217E+00	.322E+00
53	.410E-03	.507E-02	.217E-01	.582E-01	.119E+00	.202E+00	.302E+00
54	.359E-03	.450E-02	.195E-01	.529E-01	.109E+00	.187E+00	.283E+00
55	.315E-03	.400E-02	.176E-01	.481E-01	.100E+00	.174E+00	.265E+00
56	.276E-03	.356E-02	.158E-01	.438E-01	.921E-01	.161E+00	.248E+00
57	.243E-03	.318E-02	.143E-01	.399E-01	.846E-01	.150E+00	.232E+00
58	.215E-03	.284E-02	.129E-01	.364E-01	.779E-01	.139E+00	.218E+00
59	.189E-03	.254E-02	.116E-01	.331E-01	.716E-01	.129E+00	.204E+00
60	.168E-03	.227E-02	.105E-01	.302E-01	.659E-01	.120E+00	.190E+00
61	.149E-03	.203E-02	.951E-02	.276E-01	.607E-01	.111E+00	.178E+00
62	.132E-03	.182E-02	.861E-02	.252E-01	.559E-01	.103E+00	.167E+00
63	.117E-03	.164E-02	.780E-02	.230E-01	.515E-01	.956E-01	.156E+00
64	.104E-03	.147E-02	.708E-02	.211E-01	.474E-01	.887E-01	.146E+00
65	.929E-04	.132E-02	.642E-02	.193E-01	.437E-01	.824E-01	.136E+00
66	.829E-04	.119E-02	.584E-02	.176E-01	.403E-01	.765E-01	.127E+00
67	.741E-04	.108E-02	.531E-02	.162E-01	.372E-01	.711E-01	.119E+00
68	.663E-04	.972E-03	.483E-02	.148E-01	.343E-01	.660E-01	.111E+00
69	.594E-04	.879E-03	.440E-02	.136E-01	.317E-01	.613E-01	.104E+00
70	.533E-04	.795E-03	.401E-02	.125E-01	.293E-01	.570E-01	.972E-01
71	.479E-04	.720E-03	.366E-02	.115E-01	.270E-01	.530E-01	.909E-01
72	.431E-04	.653E-03	.334E-02	.105E-01	.250E-01	.493E-01	.851E-01
73	.388E-04	.593E-03	.305E-02	.967E-02	.231E-01	.459E-01	.796E-01

TABLE 2 (Cont.)

N	F=0	F=1	F=2	F=3	F=4	F=5	F=6
74	.350E-04	.539E-03	.279E-02	.890E-02	.214E-01	.427E-01	.744E-01
75	.316E-04	.490E-03	.255E-02	.819E-02	.198E-01	.397E-01	.697E-01
76	.286E-04	.446E-03	.234E-02	.755E-02	.184E-01	.370E-01	.652E-01
77	.258E-04	.406E-03	.214E-02	.696E-02	.170E-01	.345E-01	.610E-01
78	.234E-04	.370E-03	.197E-02	.642E-02	.158E-01	.321E-01	.572E-01
79	.212E-04	.338E-03	.180E-02	.592E-02	.146E-01	.299E-01	.535E-01
80	.193E-04	.309E-03	.166E-02	.547E-02	.136E-01	.279E-01	.501E-01
81	.175E-04	.282E-03	.152E-02	.505E-02	.126E-01	.260E-01	.470E-01
82	.159E-04	.258E-03	.140E-02	.467E-02	.117E-01	.243E-01	.440E-01
83	.144E-04	.236E-03	.129E-02	.432E-02	.109E-01	.227E-01	.413E-01
84	.132E-04	.217E-03	.119E-02	.400E-02	.101E-01	.212E-01	.387E-01
85	.120E-04	.199E-03	.110E-02	.370E-02	.940E-02	.198E-01	.363E-01
86	.110E-04	.182E-03	.101E-02	.343E-02	.875E-02	.185E-01	.341E-01
87	.100E-04	.168E-03	.933E-03	.318E-02	.815E-02	.173E-01	.320E-01
88	.918E-05	.154E-03	.861E-03	.295E-02	.759E-02	.161E-01	.300E-01
89	.840E-05	.142E-03	.796E-03	.274E-02	.707E-02	.151E-01	.282E-01
90	.769E-05	.130E-03	.736E-03	.254E-02	.659E-02	.141E-01	.265E-01
91	.703E-05	.120E-03	.681E-03	.236E-02	.615E-02	.132E-01	.249E-01
92	.644E-05	.111E-03	.631E-03	.219E-02	.573E-02	.124E-01	.234E-01
93	.590E-05	.102E-03	.584E-03	.204E-02	.535E-02	.116E-01	.220E-01
94	.542E-05	.944E-04	.542E-03	.190E-02	.500E-02	.109E-01	.206E-01
95	.501E-05	.872E-04	.502E-03	.177E-02	.467E-02	.102E-01	.194E-01
96	.459E-05	.806E-04	.466E-03	.165E-02	.437E-02	.956E-02	.183E-01
97	.423E-05	.746E-04	.433E-03	.153E-02	.408E-02	.897E-02	.172E-01
98	.393E-05	.690E-04	.402E-03	.143E-02	.382E-02	.842E-02	.162E-01
99	.364E-05	.640E-04	.374E-03	.134E-02	.357E-02	.790E-02	.152E-01
100	.334E-05	.592E-04	.348E-03	.125E-02	.335E-02	.742E-02	.144E-01
101	.310E-05	.550E-04	.324E-03	.116E-02	.313E-02	.697E-02	.135E-01
102	.286E-05	.510E-04	.302E-03	.109E-02	.294E-02	.655E-02	.127E-01
103	.262E-05	.474E-04	.281E-03	.102E-02	.275E-02	.616E-02	.120E-01
104	.244E-05	.440E-04	.262E-03	.950E-03	.258E-02	.579E-02	.113E-01
105	.226E-05	.409E-04	.244E-03	.889E-03	.242E-02	.545E-02	.107E-01
106	.209E-05	.380E-04	.228E-03	.832E-03	.227E-02	.513E-02	.101E-01
107	.197E-05	.354E-04	.213E-03	.779E-03	.213E-02	.483E-02	.952E-02
108	.179E-05	.330E-04	.199E-03	.729E-03	.200E-02	.455E-02	.899E-02
109	.167E-05	.307E-04	.186E-03	.684E-03	.188E-02	.428E-02	.849E-02
110	.155E-05	.287E-04	.174E-03	.641E-03	.177E-02	.404E-02	.802E-02
111	.143E-05	.267E-04	.162E-03	.601E-03	.166E-02	.381E-02	.758E-02
112	.131E-05	.249E-04	.152E-03	.564E-03	.157E-02	.359E-02	.717E-02
113	.125E-05	.232E-04	.142E-03	.529E-03	.147E-02	.339E-02	.678E-02
114	.113E-05	.218E-04	.133E-03	.497E-03	.139E-02	.319E-02	.641E-02
115	.107E-05	.203E-04	.125E-03	.467E-03	.131E-02	.302E-02	.606E-02
116	.101E-05	.190E-04	.117E-03	.439E-03	.123E-02	.285E-02	.574E-02
117	.954E-06	.178E-04	.110E-03	.413E-03	.116E-02	.269E-02	.543E-02
118	.894E-06	.166E-04	.103E-03	.388E-03	.109E-02	.254E-02	.514E-02
119	.834E-06	.156E-04	.968E-04	.365E-03	.103E-02	.240E-02	.487E-02

TABLE 2 (Cont.)

N	F=0	F=1	F=2	F=3	F=4	F=5	F=6
120	.775E-06	.146E-04	.909E-04	.344E-03	.973E-03	.227E-02	.461E-02
121	.715E-06	.136E-04	.854E-04	.324E-03	.918E-03	.215E-02	.437E-02
122	.656E-06	.128E-04	.803E-04	.305E-03	.867E-03	.203E-02	.415E-02
123	.656E-06	.120E-04	.755E-04	.288E-03	.819E-03	.192E-02	.393E-02
124	.596E-06	.113E-04	.710E-04	.271E-03	.773E-03	.182E-02	.373E-02
125	.536E-06	.106E-04	.668E-04	.256E-03	.731E-03	.172E-02	.354E-02
126	.536E-06	.995E-05	.629E-04	.241E-03	.691E-03	.163E-02	.336E-02
127	.477E-06	.936E-05	.592E-04	.228E-03	.653E-03	.155E-02	.318E-02
128	.477E-06	.882E-05	.558E-04	.215E-03	.618E-03	.147E-02	.302E-02
129	.417E-06	.829E-05	.526E-04	.203E-03	.585E-03	.139E-02	.287E-02
130	.417E-06	.781E-05	.496E-04	.192E-03	.554E-03	.132E-02	.273E-02
131	.358E-06	.733E-05	.468E-04	.181E-03	.524E-03	.125E-02	.259E-02
132	.358E-06	.691E-05	.442E-04	.171E-03	.496E-03	.119E-02	.246E-02
133	.358E-06	.650E-05	.417E-04	.162E-03	.470E-03	.112E-02	.234E-02
134	.298E-06	.614E-05	.393E-04	.153E-03	.446E-03	.107E-02	.223E-02
135	.298E-06	.578E-05	.372E-04	.145E-03	.422E-03	.101E-02	.212E-02
136	.298E-06	.542E-05	.351E-04	.137E-03	.401E-03	.963E-03	.201E-02
137	.238E-06	.513E-05	.332E-04	.130E-03	.380E-03	.915E-03	.192E-02
138	.238E-06	.483E-05	.314E-04	.123E-03	.360E-03	.869E-03	.182E-02
139	.238E-06	.459E-05	.297E-04	.117E-03	.342E-03	.826E-03	.174E-02
140	.238E-06	.435E-05	.281E-04	.111E-03	.325E-03	.786E-03	.165E-02
141	.179E-06	.411E-05	.266E-04	.105E-03	.308E-03	.747E-03	.158E-02
142	.179E-06	.387E-05	.252E-04	.994E-04	.293E-03	.711E-03	.150E-02
143	.179E-06	.364E-05	.238E-04	.944E-04	.278E-03	.676E-03	.143E-02
144	.179E-06	.346E-05	.226E-04	.895E-04	.265E-03	.644E-03	.136E-02
145	.179E-06	.328E-05	.214E-04	.850E-04	.252E-03	.613E-03	.130E-02
146	.179E-06	.310E-05	.203E-04	.807E-04	.239E-03	.584E-03	.124E-02
147	.119E-06	.292E-05	.193E-04	.767E-04	.227E-03	.556E-03	.118E-02
148	.119E-06	.280E-05	.183E-04	.728E-04	.216E-03	.530E-03	.113E-02
149	.119E-06	.262E-05	.173E-04	.692E-04	.206E-03	.505E-03	.108E-02
150	.119E-06	.250E-05	.165E-04	.657E-04	.196E-03	.481E-03	.103E-02

TABLE 3

UNAVAILABILITY FOR LARGE LOSS OF COOLANT ACCIDENTS
(99% confidence)

Number of Panels installed per reactor building = 29
Number of Panels required to burst per reactor building = 10

N	F=0	F=1	F=2	F=3	F=4	F=5	F=6
10	.458E-03	.341E-01	.254E+00	.649E+00	.918E+00	.992E+00	.100E+01
11	.142E-03	.141E-01	.137E+00	.454E+00	.792E+00	.959E+00	.997E+00
12	.459E-04	.578E-02	.709E-01	.293E+00	.625E+00	.878E+00	.979E+00
13	.156E-04	.240E-02	.356E-01	.178E+00	.456E+00	.751E+00	.929E+00
14	.554E-05	.101E-02	.177E-01	.104E+00	.313E+00	.600E+00	.838E+00
15	.209E-05	.437E-03	.874E-02	.589E-01	.205E+00	.451E+00	.713E+00
16	.775E-06	.193E-03	.434E-02	.330E-01	.130E+00	.323E+00	.575E+00
17	.298E-06	.868E-04	.217E-02	.183E-01	.801E-01	.223E+00	.442E+00
18	.119E-06	.401E-04	.110E-02	.101E-01	.488E-01	.149E+00	.327E+00
19	.596E-07	.189E-04	.562E-03	.564E-02	.294E-01	.982E-01	.234E+00
20	.000E+00	.912E-05	.292E-03	.314E-02	.177E-01	.636E-01	.164E+00
21	.000E+00	.453E-05	.154E-03	.177E-02	.106E-01	.408E-01	.113E+00
22	.000E+00	.226E-05	.822E-04	.100E-02	.636E-02	.260E-01	.767E-01
23	.000E+00	.119E-05	.446E-04	.571E-03	.383E-02	.165E-01	.516E-01
24	.000E+00	.596E-06	.245E-04	.330E-03	.232E-02	.105E-01	.345E-01
25	.000E+00	.298E-06	.137E-04	.192E-03	.141E-02	.669E-02	.230E-01
26	.000E+00	.179E-06	.775E-05	.113E-03	.864E-03	.427E-02	.153E-01
27	.000E+00	.119E-06	.447E-05	.672E-04	.533E-03	.273E-02	.102E-01
28	.000E+00	.596E-07	.256E-05	.404E-04	.331E-03	.175E-02	.677E-02
29	.000E+00	.596E-07	.149E-05	.245E-04	.207E-03	.113E-02	.451E-02
30	.000E+00	.000E+00	.894E-06	.150E-04	.131E-03	.736E-03	.302E-02
31	.000E+00	.000E+00	.536E-06	.930E-05	.831E-04	.480E-03	.202E-02
32	.000E+00	.000E+00	.358E-06	.584E-05	.532E-04	.315E-03	.136E-02
33	.000E+00	.000E+00	.179E-06	.370E-05	.343E-04	.208E-03	.922E-03
34	.000E+00	.000E+00	.119E-06	.232E-05	.224E-04	.139E-03	.627E-03
35	.000E+00	.000E+00	.596E-07	.149E-05	.147E-04	.926E-04	.428E-03
36	.000E+00	.000E+00	.596E-07	.954E-06	.966E-05	.623E-04	.293E-03
37	.000E+00	.000E+00	.596E-07	.656E-06	.644E-05	.421E-04	.202E-03
38	.000E+00	.000E+00	.000E+00	.417E-06	.429E-05	.287E-04	.140E-03
39	.000E+00	.000E+00	.000E+00	.298E-06	.292E-05	.197E-04	.973E-04
40	.000E+00	.000E+00	.000E+00	.179E-06	.197E-05	.135E-04	.681E-04
41	.000E+00	.000E+00	.000E+00	.119E-06	.137E-05	.936E-05	.478E-04
42	.000E+00	.000E+00	.000E+00	.596E-07	.954E-06	.656E-05	.337E-04
43	.000E+00	.000E+00	.000E+00	.596E-07	.656E-06	.459E-05	.240E-04
44	.000E+00	.000E+00	.000E+00	.596E-07	.417E-06	.322E-05	.171E-04
45	.000E+00	.000E+00	.000E+00	.000E+00	.298E-06	.226E-05	.122E-04
46	.000E+00	.000E+00	.000E+00	.000E+00	.238E-06	.161E-05	.882E-05
47	.000E+00	.000E+00	.000E+00	.000E+00	.179E-06	.119E-05	.638E-05
48	.000E+00	.000E+00	.000E+00	.000E+00	.119E-06	.834E-06	.465E-05
49	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.596E-06	.340E-05

TABLE 3 (Cont.)

N	F=0	F=1	F=2	F=3	F=4	F=5	F=6
50	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.417E-06	.244E-05
51	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.298E-06	.179E-05
52	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.238E-06	.137E-05
53	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.179E-06	.101E-05
54	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.119E-06	.715E-06
55	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.119E-06	.536E-06
56	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.417E-06
57	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.298E-06
58	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.238E-06
59	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07	.179E-06
60	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.119E-06
61	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.119E-06
62	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07
63	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07
64	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07
65	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.596E-07
66	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00

APPENDIX A

BAYESIAN APPROACH

1. Introduction

In estimating the probability of failure of a device, prior information about the quality of the device may be available, either qualitatively or quantitatively. For example, the design of the pressure relief panels and the reputation of the manufacturer is such that a good quality on the panels can reasonably be expected. It is useful to have a method to incorporate this information into the analysis. The Bayesian approach/1/ provides the required methodology.

In the Bayesian approach the parameters of interest (like the probability of failure of a panel, p) are considered to be random variables. The initial (prior) distribution of the random variables is selected to reflect the known information of the parameters. As data becomes available, the distribution of the parameters is updated (using the Bayes theorem/1/) to incorporate the new information. At any stage, interval estimates of the parameter can be obtained from the distribution.

2. Prior Information

The distribution function selected to represent the parameters of interest is very important. First, it should allow to represent the qualitative information available for the parameters, and second, it should include any restrictions on the values of the parameters. For the parameter p these can be achieved by selecting its distribution from the family of Beta(α,β) distributions/1,2/. This is a family of continuous distributions, restricted to the interval [0,1], and whose members are completely specified by giving the values of α and β. Figure A.1, taken from /3/, gives some examples of this distribution.

To see how prior information about the quality of the panels can be quantified, suppose that the panels are of good quality. This implies that the probability distribution function for p should be concentrated on the left hand side of the interval [0,1]. So, a Beta(1,2), or a Beta(0.5,2), or another distribution with similar shape, can be selected as the prior distribution for p. If it is not possible to make any prior inference about the quality of the panels, then the uniform prior distribution (i.e., a Beta(1,1) which gives the same likelihood to all values) can be selected.

3. Posterior Distribution

When sample data is obtained, the prior distribution needs to be updated. In the case of p it is very simple, for if N panels are tested and F are found to be defective, the posterior distribution for p will be a Beta($F+\alpha$, $N-F+\beta$). For example, suppose an uniform prior distribution for p is selected and that of 50 panels tested no defective is found. Then, the posterior distribution will be a Beta(1, 51).

4. Interval Estimates

The posterior distribution of the parameter gives all the information contained in the data and the prior information. However, in many cases it is convenient to give numerical values for a few percentage points. These percentage points act as interval estimates for the parameter. For ex-

ample, Table A.1 gives values of the 50%, 75%, 95%, and 99% percentage point for a Beta(1,51); these can be compared with those given in Table 1.

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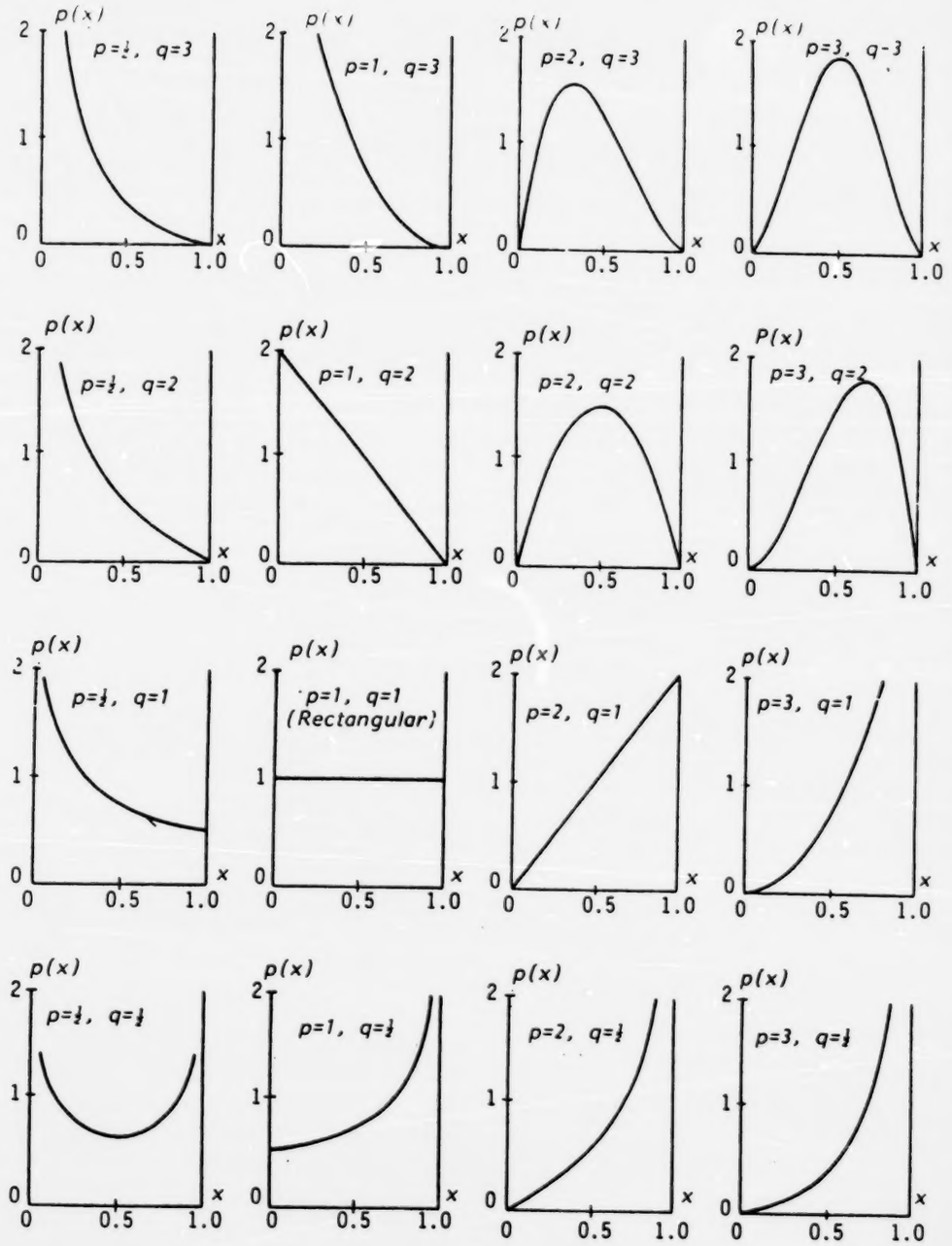

FIGURE 1a
BETA DENSITY FUNCTION β(p-q)

TABLE A.1

Bayesian Interval Estimates for the Probability Failure of the Panels and Probability that at least 21 out of 29 panels operate

Prior Distribution :	BETA(1,1)
Number of Panels Initially Tested :	60
Number of Failures :	0
Number of Panels installed per Reactor Building :	29

Percentile Point	UNAV(p) Per Reactor	UNAV(p) All Reactors	p
Expected value	.128E-04	.513E-04	.155E-01
50% probability	.000	.000	.126E-01
95% probability	.560E-05	.224E-04	.492E-01
98% probability	.441E-04	.176E-03	.634E-01
99% probability	.149E-03	.595E-03	.740E-01

APPENDIX B

CALCULATION OF THE AVAILABILITY USING NORMALITY ASSUMPTION

1. Introduction

The method given in Section 2 of the report uses Equations 1 and 2 to estimate the availability and unavailability of the pressure relief system by replacing the estimate $\hat{p}$ or p_u for p. The estimates are obtained by counting the number of failures F in a set of N trials. Better estimates for p can be obtained if it can be assumed that the rupture pressure values of the panels have a normal distribution. This assumption can be tested in light of the data.

To outline the methodology, suppose the pressure breaking point of a panel, X, has a Normal distribution with mean μ and variance σ^2; this is denoted by $X \sim N(\mu,\sigma^2)$. Also, assume that a panel is consider defective if its bursting pressure is outside the interval 15 ± 2.5 kpa. If the parameter μ and σ^2 were known, p would be given by

$$p = 1 - \text{Prob}\{ 12.5 \le X \le 17.5 \}$$
$$= 1 - \int_{12.5}^{17.5} f(x)\, dx,$$

where $f(x)$ denotes the probability density function. This value of p could be substituted in Equations 1 and 2 to obtain the values of the availability and unavailability of the system.

2. Estimation

Usually, the parameters have to be estimated from a sample of N panels. The sample pressure mean, $\bar{x}$, and the sample variance, s^2, are calculated from the sample results, x_i, i = 1, ..., N, as

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$
$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2.$$

If U = 15+2.5, L = 15-2.5, and $\beta(N/2-1)$ denote a Beta($N/2$ - 1,$N/2$ - 1), p can be estimated by/1/

$$\hat{p} = \int_0^{\max[0,\ 1/2 - (U-x)\sqrt{N}/2s(N-1)]} d\beta(N/2 - 1) +$$
$$\int_0^{\max[0,\ 1/2 - (x-L)\sqrt{N}/2s(N-1)]} d\beta(N/2 - 1)$$

To obtain an α confidence interval, the following procedure is used. Let $p^* = p$. Then, find β_p^*, k, δ a defined by the following equations.

$$p^* = \int_0^{\beta_p^*} d\beta(N/2 - 1)$$

$$\sqrt{N}\,k = (n-1)(1-2\beta_p^*)$$

$$\alpha = \Pr\{ t_{N-1,\delta} \geq \sqrt{N}\,k\}$$

$$p_0 = \int_{\delta/\sqrt{N}}^{\infty} Z(x)dx$$

where $t_{N-1,\delta}$ represent a non-central student t distribution with N-1 degrees of freedom and noncentrality parameter δ, and $Z(x)$ is the probability density function of the standard normal distribution i.e., a $N(0,1)$. The confidence interval is given by $[0, p_0]$. The value of p_0 can be substituted in Equations 1 and 2 of the report to obtain confidence intervals for the availability and unavailability of the pressure relief system.

3. Discussion

It can be shown that when the distribution of the pressure breaking points of the panels is Normal, the method described in this appendix gives more accurate estimates than the one described in the report. However, if this is not the case, do not use this method. Fortunately, the normality assumption can be tested with the sampled pressure data using the Shapiro and Wilk/2/ method implemented in SAS/3/. If the test is not rejected, the method described in this appendix can be used.

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